

	Example 1
	$\{1,2\}\succ_1\{1\}\succ_1\{1,2,3\}\succ_1\{1,3\}$
	$\{1,2\}\succ_2\{2\}\succ_2\{1,2,3\}\succ_2\{2,3\}$
	$\{1,2,3\} \succ_3 \{2,3\} \succ_3 \{1,3\} \succ_3 \{3\}$
	in the core and is individually stable. Nash stable partitions.
$\{\!\{1\},\!\{2\},\!\{3\}\!\}$	{1,2} is preferred by both agent 1 and 2, hence not NS, not IS.
{{1,2},{3}}	{1,2,3} is preferred by agent 3, so it is not NS, as agents 1 and 3 are worse off, it is not a possible move for IS. no other move is possible for IS.
{{1,3},{2}}	agent 1 prefers to be on its own (not NS, then, not IS).
$\{\{2,3\},\{1\}\}$	agent 2 prefers to join agent 1, and agent 1 is better off, hence not NS, not IS.
{{1,2,3}}	agents 1 and 2 have an incentive to form a singleton, hence not NS, not IS.

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Example 3

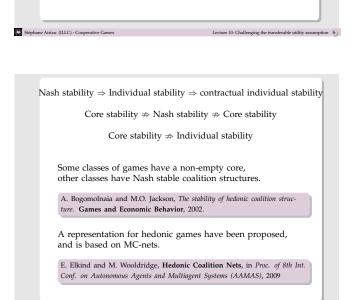
 $\{1,2\} \succ_1 \{1,3\} \succ_1 \{1\} \succ_1 \{1,2,3\} \\ \{2,3\} \succ_2 \{1,2\} \succ_2 \{2\} \succ_2 \{1,2,3\}$

 $\{1,3\}\succ_3\{2,3\}\succ_3\{3\}\succ_3\{1,2,3\}$ The core is empty (similar argument as for example 2).

There is no Nash stable partition or individually stable partition. But there are three contractually individually stable CSs: $\{\{1,2\},\{3\}\},\{\{1,3\},\{2\}\},\{\{2,3\},\{1\}\}\}$.

For {{1,2},{3}}:

- {{1},{2,3}}: agents 2 and 3 benefit, hence {{1,2},{3}} is not Nash or individually stable. however, agent 1 is worse off, hence not a possible move for CIS.
- {{2},{1,3}}: agent 1 has no incentive to join agent 3.
- {[1], {2], {3}}: neither agent 1 or 2 has any incentive to form a
- singleton coalition.



Example 2

$$\begin{split} &\{1,2\}\succ_1 \{1,3\}\succ_1 \{1,2,3\}\succ_1 \{1\} \\ &\{2,3\}\succ_2 \{1,2\}\succ_2 \{1,2,3\}\succ_2 \{2\} \\ &\{1,3\}\succ_3 \{2,3\}\succ_3 \{1,2,3\}\succ_3 \{3\} \end{split}$$

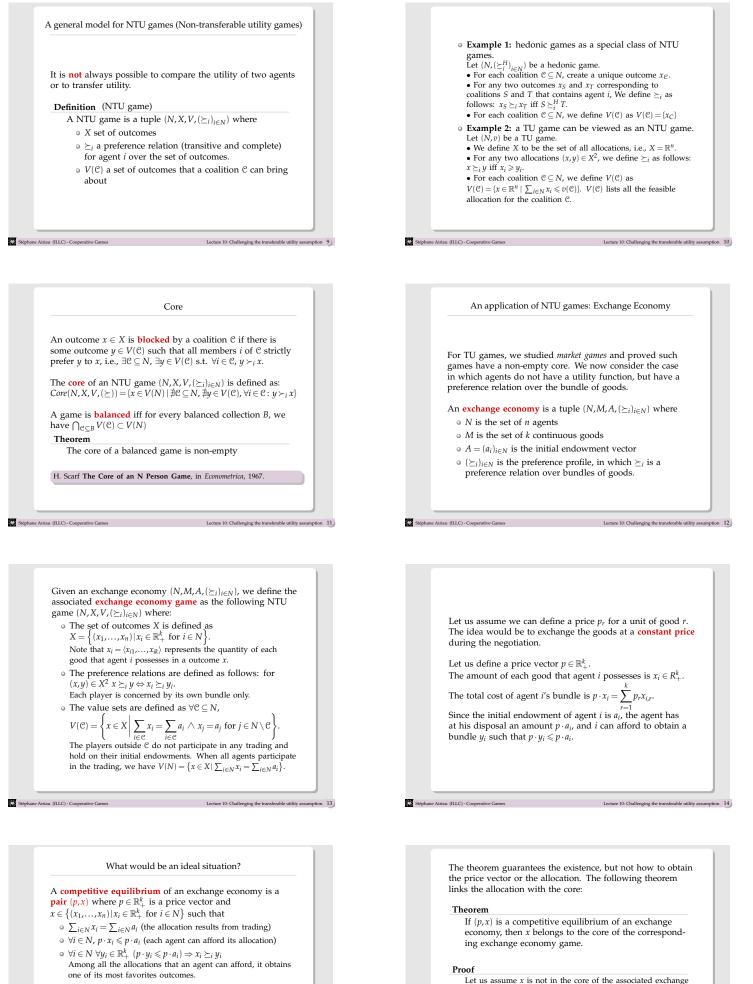
 $\{1,2\}, \{1,3\}, \{2,3\}$ and $\{1,2,3\}$ are blocking $\{2,3\}$ is blocking

{1,2} is blocking

The core is empty $\{\{1\},\{2\},\{3\}\}$ $\{1,2\},$ $\{\{1,2\},\{3\}\}$ $\{2,3\}$

the grand coalition).

{{1,3},{2}}



Using the price vector and the allocation, each agent believes it possesses the best outcome.

Theorem

Let $(N,M,A,(\succeq_i)_{i \in N})$ be an exchange economy. If each preference relation \succeq_i is continuous and strictly convex, then a competitive equilibrium exists.

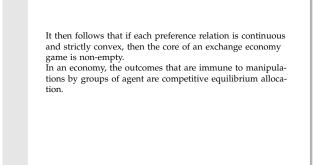
which is a contradiction.

economy game. Then, there is at least one coalition $\ensuremath{\mathbb{C}}$ and

an allocation *y* such that $\forall i \in C \ y \succ_i x$. By definition of the competitive equilibrium, we must have $p \cdot y_i > p \cdot a_i$. Summing

the prices are positive, we deduce that $\sum_{i \in \mathcal{C}} y_i > \sum_{i \in \mathcal{C}} a_i$,

over all the agents in C, we have $p \cdot \sum_{i \in \mathbb{C}} y_i > p \cdot \sum_{i \in \mathbb{C}} a_i$. Since



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Coming next
• Deriving cooperative games from non-cooperative one

We considered Hedonic games, an example of games in which utility cannot be transferred between agents.
We defined general NTU games
We studied an important application of NTU games: the exchange economy.

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